
Contents

Part I Green's Functions in Mathematical Physics

1 Time-Independent Green's Functions	3
1.1 Formalism	3
1.2 Examples	9
1.2.1 Three-Dimensional Case ($d = 3$)	10
1.2.2 Two-Dimensional Case ($d = 2$)	11
1.2.3 One-Dimensional Case ($d = 1$)	13
1.2.4 Finite Domain Ω	13
1.3 Summary	14
1.3.1 Definition	14
1.3.2 Basic Properties	15
1.3.3 Methods of Calculation	15
1.3.4 Use	16
Further Reading	17
Problems	17
2 Time-Dependent Green's Functions	21
2.1 First-Order Case	21
2.1.1 Examples	24
2.2 Second-Order Case	26
2.2.1 Examples	30
2.3 Summary	33
2.3.1 Definition	33
2.3.2 Basic Properties	33
2.3.3 Definition	34
2.3.4 Basic Properties	34
2.3.5 Use	35
Further Reading	36
Problems	36

Part II Green's Functions in One-Body Quantum Problems

3 Physical Significance of G.	41
Application to the Free-Particle Case	41
3.1 General Relations	41
3.2 The Free-Particle ($\mathcal{H}_0 = p^2/2m$) Case	43
3.2.1 3-d Case	44
3.2.2 2-d Case	45
3.2.3 1-d Case	45
3.3 The Free-Particle Klein–Gordon Case	47
3.4 Summary	50
Further Reading	51
Problems	51
4 Green's Functions and Perturbation Theory	55
4.1 Formalism	55
4.1.1 Time-Independent Case	55
4.1.2 Time-Dependent Case	60
4.2 Applications	64
4.2.1 Scattering Theory ($E > 0$)	64
4.2.2 Bound State in Shallow Potential Wells ($E < 0$)	67
4.2.3 The KKR Method for Electronic Calculations in Solids	70
4.3 Summary	71
Further Reading	74
Problems	74
5 Green's Functions for Tight-Binding Hamiltonians	77
5.1 Introductory Remarks	77
5.2 The Tight-Binding Hamiltonian (TBH)	80
5.3 Green's Functions	87
5.3.1 One-Dimensional Lattice	88
5.3.2 Square Lattice	89
5.3.3 Simple Cubic Lattice	94
5.3.4 Green's Functions for Bethe Lattices (Cayley Trees)	98
5.4 Summary	101
Further Reading	102
Problems	102
6 Single Impurity Scattering	111
6.1 Formalism	111
6.2 Explicit Results for a Single Band	118
6.2.1 Three-Dimensional Case	118
6.2.2 Two-Dimensional Case	123
6.2.3 One-Dimensional Case	124

6.3	Applications	125
6.3.1	Levels in the Gap	125
6.3.2	The Cooper Pair and Superconductivity	127
6.3.3	The Kondo Problem	133
6.3.4	Lattice Vibrations in Crystals Containing “Isotope” Impurities	135
6.4	Summary	137
	Further Reading	139
	Problems	140
7	Two or More Impurities; Disordered Systems	141
7.1	Two Impurities	141
7.2	Infinite Number of Impurities	150
7.2.1	Virtual Crystal Approximation (VCA)	151
7.2.2	Average t -Matrix Approximation (ATA)	152
7.2.3	Coherent Potential Approximation (CPA)	153
7.2.4	The CPA for Classical Waves	158
7.2.5	Direct Extensions of the CPA	163
7.2.6	Cluster Generalizations of the CPA	165
7.3	Summary	168
	Further Reading	169
	Problems	169
8	Electrical Conductivity and Green’s Functions	173
8.1	Electrical Conductivity and Related Quantities	173
8.2	Various Methods of Calculation	176
8.2.1	Phenomenological Approach	176
8.2.2	Boltzmann’s Equation	177
8.2.3	A General, Independent-Particle Formula for Conductivity	178
8.2.4	General Linear Response Theory	180
8.3	Conductivity in Terms of Green’s Functions	183
8.3.1	Conductivity Without Vertex Corrections	184
8.3.2	CPA for Vertex Corrections	186
8.3.3	Vertex Corrections Beyond the CPA	190
8.3.4	Post-CPA Corrections to Conductivity	192
8.4	Summary	195
	Further Reading	197
	Problems	197
9	Localization, Transport, and Green’s Functions	199
9.1	An Overview	199
9.2	Disorder, Diffusion, and Interference	203
9.3	Localization	208
9.3.1	Three-Dimensional Systems	210

9.3.2	Two-Dimensional Systems	212
9.3.3	One-Dimensional and Quasi-One-Dimensional Systems	214
9.4	Conductance and Transmission	216
9.5	Scaling Approach	219
9.6	Other Calculational Techniques	224
9.6.1	Quasi-One-Dimensional Systems and Scaling	225
9.6.2	Level Spacing Statistics	225
9.7	Localization and Green's Functions	226
9.7.1	Green's Function and Localization in One Dimension	227
9.7.2	Renormalized Perturbation Expansion (RPE) and Localization	230
9.7.3	Green's Functions and Transmissions in Quasi-One-Dimensional Systems	235
9.8	Applications	238
9.9	Summary	240
	Further Reading	243
	Problems	243

Part III Green's Functions in Many-Body Systems

10	Definitions	249
10.1	Single-Particle Green's Functions in Terms of Field Operators	249
10.2	Green's Functions for Interacting Particles	253
10.3	Green's Functions for Noninteracting Particles	257
10.4	Summary	260
	Further Reading	261
	Problems	261
11	Properties and Use of the Green's Functions	263
11.1	Analytical Properties of g_s and \tilde{g}_s	263
11.2	Physical Significance and Use of g_s and \tilde{g}_s	268
11.3	Quasiparticles	275
11.4	Summary	281
11.4.1	Properties	281
11.4.2	Use	282
	Further Reading	283
	Problems	283
12	Calculational Methods for g	285
12.1	Equation of Motion Method	285
12.2	Diagrammatic Method for Fermions at $T = 0$	289
12.3	Diagrammatic Method for $T \neq 0$	298
12.4	Partial Summations. Dyson's Equation	300
12.5	Other Methods of Calculation	306

12.6 Summary	306
Further Reading	307
Problems	307
13 Applications	309
13.1 Normal Fermi Systems. Landau Theory	309
13.2 High-Density Electron Gas	312
13.3 Dilute Fermi Gas	319
13.4 Superconductivity	322
13.4.1 Diagrammatic Approach	322
13.4.2 Equation of Motion Approach	324
13.5 The Hubbard Model	327
13.6 Summary	332
Further Reading	333
Problems	334
A Dirac's delta Function	337
B Dirac's bra and ket Notation	341
C Solutions of Laplace and Helmholtz Equations in Various Coordinate Systems	345
C.1 Helmholtz Equation $(\nabla^2 + k^2) \psi(\mathbf{r}) = 0$	345
C.1.1 Cartesian Coordinates x, y, z	345
C.1.2 Cylindrical Coordinates z, ϕ, ϱ	345
C.1.3 Spherical coordinates r, θ, ϕ	346
C.2 Vector Derivatives	347
C.2.1 Spherical Coordinates r, θ, ϕ	347
C.2.2 Cylindrical Coordinates z, ϱ, ϕ	348
C.3 Schrödinger Equation in Centrally Symmetric 3- and 2-Dimensional Potential V	348
D Analytic Behavior of $G(z)$ Near a Band Edge	351
E Wannier Functions	355
F Renormalized Perturbation Expansion (RPE)	357
G Boltzmann's Equation	363
H Transfer Matrix, S-Matrix, etc.	369
I Second Quantization	379
Solutions of Selected Problems	391
References	447
Index	461