Contents

Preface ix

Introduction 1

Part I Geometry

1 The Klein correspondence 7
- 1.1 Projective space and Grassmannian manifolds 7
- 1.2 Twistor manifolds and correspondences 19
- 1.3 The Plücker embedding of $\mathbf{M}$ into $\mathbf{P}_5$ 26
- 1.4 The linear geometry of the Klein quadric 28
- 1.5 Group actions and homogeneous structures 36
- 1.6 Minkowski and Euclidean space 45

2 Fiber bundles 68
- 2.1 Vector bundles and principal bundles 68
- 2.2 Differential forms 92
- 2.3 Tensor and spinor bundles 115
- 2.4 Connections and curvature 131

3 The algebraic topology of manifolds and bundles 148
- 3.1 Homotopy and homology 149
- 3.2 Sheaf theory 166
- 3.3 Sheaf cohomology 175
- 3.4 Characteristic classes 197
- 3.5 Clifford algebras and spin bundles 208
- 3.6 Spectral sequences 218

Part II Classical field theory

4 Linear field theories 241
- 4.1 The wave equation and Maxwell's equations 241
- 4.2 Spinors and spinor fields 245
- 4.3 The action principle and interactions 253
- 4.4 Poincaré and conformal invariance 258

5 Gauge theory 263
- 5.1 The essentials of gauge theory 264

5.2 Yang–Mills instantons 270
5.3 Magnetic poles 279

6 General relativity 286
6.1 Space-time, spinors, and Einstein's equations 286
6.2 Self-duality and gravitational instantons 293

Part III The Penrose transform

7 Massless free fields 301
7.1 Holomorphic solutions of the massless field equations 303
7.2 The linear Penrose transform 325
7.3 Integral formulas for massless fields 349
7.4 Hyperfunction solutions of the massless field equations 354

8 Self-dual gauge fields 371
8.1 Correspondence between self-dual gauge fields and holomorphic bundles 372
8.2 Ansätze for SU(2)-fields 398
8.3 The twistor construction of instantons 410
8.4 Magnetic poles: solutions 422

9 Twistors for self-dual space-time 433
9.1 Correspondence between self-dual space-times and curved twistor spaces 434
9.2 Constructing self-dual space-times 440

10 The Penrose transform for general gauge fields 459
10.1 The Penrose transformation on formal neighborhoods of the space of null lines 460
10.2 Supergeometry and Yang–Mills fields 472

References 485
Subject and author index 504