Contents

Preface to the Third Edition vii

Preface to the Second Edition ix

Preface to the First Edition xi

Notation xxiii

Chapter 1 | Introduction 1

1. Elasticity 1
2. Stress 2
3. Notation for Forces and Stresses 3
4. Components of Stress 4
5. Components of Strain 6
6. Hooke's Law 8
7. Index Notation 12

Problems 14

Chapter 2 | Plane Stress and Plane Strain 15

8. Plane Stress 15
9. Plane Strain 15
10. Stress at a Point 17
11. Strain at a Point 23
12. Measurement of Surface Strains 24
13. Construction of Mohr Strain Circle for Strain Rosette 26

14. Differential Equations of Equilibrium **26**
15. Boundary Conditions **28**
16. Compatibility Equations **29**
17. Stress Function **31**
Problems **33**

Chapter 3 | Two-dimensional Problems in Rectangular Coordinates 35

18. Solution by Polynomials **35**
19. End Effects. Saint-Venant's Principle **39**
20. Determination of Displacements **40**
21. Bending of a Cantilever Loaded at the End **41**
22. Bending of a Beam by Uniform Load **46**
23. Other Cases of Continuously Loaded Beams **50**
24. Solution of the Two-dimensional Problem in the Form of a Fourier Series **53**
25. Other Applications of Fourier Series. Gravity Loading **60**
26. End Effects. Eigensolutions **61**
Problems **63**

Chapter 4 | Two-dimensional Problems in Polar Coordinates 65

27. General Equations in Polar Coordinates **65**
28. Stress Distribution Symmetrical about an Axis **68**
29. Pure Bending of Curved Bars **71**
30. Strain Components in Polar Coordinates **75**
31. Displacements for Symmetrical Stress Distributions **77**
32. Rotating Disks **80**
33. Bending of a Curved Bar by a Force at the End **83**
34. Edge Dislocation **88**
35. The Effect of Circular Holes on Stress Distributions in Plates **90**
36. Concentrated Force at a Point of a Straight Boundary **97**
37. Any Vertical Loading of a Straight Boundary **104**
38. Force Acting on the End of a Wedge **109**
39. Bending Couple Acting on the End of a Wedge **112**
40. Concentrated Force Acting on a Beam **113**
41. Stresses in a Circular Disk **122**
42. Force at a Point of an Infinite Plate **127**
43. Generalized Solution of the Two-dimensional Problem in Polar Coordinates **132**
44. Applications of the Generalized Solution in Polar Coordinates **136**
45. A Wedge Loaded along the Faces **139**
46. Eigensolutions for Wedges and Notches **141**
Problems **144**

Chapter 5 | Photoelastic and Moiré Experimental Methods 150

47. Experimental Methods and Verifications **150**
48. Photoelastic Stress Measurement **150**
49. Circular Polariscope **155**
50. Examples of Photoelastic Stress Determination **157**
51. Determination of the Principal Stresses **160**
52. Three-dimensional Photoelasticity **162**
53. The Moiré Method **164**

Chapter 6 | Two-dimensional Problems in Curvilinear Coordinates 168

54. Functions of a Complex Variable **168**
55. Analytic Functions and Laplace's Equation **170**
    Problems **171**
56. Stress Functions in Terms of Harmonic and Complex Functions **172**
57. Displacement Corresponding to a Given Stress Function **175**
58. Stress and Displacement in Terms of Complex Potentials **176**
59. Resultant of Stress on a Curve. Boundary Conditions **179**
60. Curvilinear Coordinates **181**
61. Stress Components in Curvilinear Coordinates **185**
    Problems **187**
62. Solutions in Elliptic Coordinates. Elliptic Hole in Uniformly Stressed Plate **187**
63. Elliptic Hole in a Plate under Simple Tension **191**
64. Hyperbolic Boundaries. Notches **194**
65. Bipolar Coordinates **196**
66. Solutions in Bipolar Coordinates **198**
    Other Curvilinear Coordinates **202**
    Assignable Shapes **203**
67. Determination of the Complex Potentials from Given Boundary Conditions. Methods of Muskhelishvili **203**
68. Formulas for the Complex Potentials **206**
69. Properties of Stress and Deformation Corresponding to Complex Potentials Analytic in the Material Region around a Hole **207**
70. Theorems on Boundary Integrals **209**
71. A Mapping Function $\omega(\xi)$ for the Elliptic Hole. The Second Boundary Integral **212**

72. The Elliptic Hole. Formula for $\psi(\xi)$ 213
73. The Elliptic Hole. Particular Problems 214
Problems 217

Chapter 7 | Analysis of Stress and Strain in Three Dimensions 219

74. Introduction 219
75. Principal Stresses 221
76. Stress Ellipsoid and Stress-director Surface 222
77. Determination of the Principal Stresses 223
78. Stress Invariants 224
79. Determination of the Maximum Shearing Stress 224
80. Homogeneous Deformation 226
81. Strain at a Point 228
82. Principal Axes of Strain 231
83. Rotation 232
Problems 234

Chapter 8 | General Theorems 235

84. Differential Equations of Equilibrium 235
85. Conditions of Compatibility 237
86. Determination of Displacements 240
87. Equations of Equilibrium in Terms of Displacements 240
88. General Solution for the Displacements 242
89. The Principal of Superposition 243
90. Strain Energy 244
91. Strain Energy of an Edge Dislocation 249
92. Principle of Virtual Work 250
93. Castigliano's Theorem 254
94. Applications of the Principle of Least Work—Rectangular Plates 258
95. Effective Width of Wide Beam Flanges 262
Problems 268
96. Uniqueness of Solution 269
97. The Reciprocal Theorem 271
98. Approximate Character of the Plane Stress Solutions 274
Problems 277

Chapter 9 | Elementary Problems of Elasticity in Three Dimensions 278

99. Uniform Stress 278
100. Stretching of a Prismatical Bar by Its Own Weight 279

101. Twist of Circular Shafts of Constant Cross Section **282**
102. Pure Bending of Prismatical Bars **284**
103. Pure Bending of Plates **288**

Chapter 10 | Torsion 291

104. Torsion of Straight Bars **291**
105. Elliptic Cross Section **297**
106. Other Elementary Solutions **299**
107. Membrane Analogy **303**
108. Torsion of a Bar of Narrow Rectangular Cross Section **307**
109. Torsion of Rectangular Bars **309**
110. Additional Results **313**
111. Solution of Torsional Problems by Energy Method **315**
112. Torsion of Rolled Profile Sections **321**
113. Experimental Analogies **324**
114. Hydrodynamical Analogies **325**
115. Torsion of Hollow Shafts **328**
116. Torsion of Thin Tubes **332**
117. Screw Dislocations **336**
118. Torsion of a Bar in Which One Cross Section Remains Plane **338**
119. Torsion of Circular Shafts of Variable Diameter **341**
Problems **349**

Chapter 11 | Bending of Bars 354

120. Bending of a Cantilever **354**
121. Stress Function **356**
122. Circular Cross Section **358**
123. Elliptic Cross Section **359**
124. Rectangular Cross Section **361**
125. Additional Results **366**
126. Nonsymmetrical Cross Sections **369**
127. Shear Center **371**
128. The Solution of Bending Problems by the Soap-film Method **374**
129. Displacements **378**
130. Further Investigations of Bending **378**

Chapter 12 | Axisymmetric Stress and Deformation in a Solid of Revolution 380

131. General Equations **380**
132. Solution by Polynomials **383**
133. Bending of a Circular Plate **385**

134. The Rotating Disk as a Three-dimensional Problem **388**
135. Force at a Point in an Infinite Solid **390**
136. Spherical Container under Internal or External Uniform Pressure **392**
137. Local Stresses around a Spherical Cavity **396**
138. Force on Boundary of a Semi-infinite Body **398**
139. Load Distributed over a Part of the Boundary of a Semi-infinite Solid **403**
140. Pressure between Two Spherical Bodies in Contact **409**
141. Pressure between Two Bodies in Contact. More General Case **414**
142. Impact of Spheres **420**
143. Symmetrical Deformation of a Circular Cylinder **422**
144. The Circular Cylinder with a Band of Pressure **425**
145. Boussinesq's Solution in Two Harmonic Functions **428**
146. The Helical Spring under Tension (Screw Dislocation in a Ring) **429**
147. Pure Bending of an Incomplete Ring **432**

Chapter 13 | Thermal Stress 433

148. The Simplest Cases of Thermal Stress Distribution. Method of Strain Suppression **433**
Problems **438**
149. Longitudinal Temperature Variation in a Strip **439**
150. The Thin Circular Disk: Temperature Symmetrical about Center **441**
151. The Long Circular Cylinder **443**
Problems **452**
152. The Sphere **452**
153. General Equations **456**
154. Thermoelastic Reciprocal Theorem **459**
155. Overall Thermoelastic Deformations. Arbitrary Temperature Distribution **460**
156. Thermoelastic Displacement. Maisel's Integral Solution **463**
Problems **466**
157. Initial Stress **466**
158. Total Volume Change Associated with Initial Stress **468**
159. Plane Strain and Plane Stress. Method of Strain Suppression **469**
160. Two-dimensional Problems with Steady Heat Flow **470**
161. Thermal Plane Stress Due to Disturbance of Uniform Heat Flow by an Insulated Hole **475**
162. Solutions of the General Equations. Thermoelastic Displacement Potential **476**
163. The General Two-dimensional Problem for Circular Regions **481**
164. The General Two-dimensional Problem in Complex Potentials **482**

Chapter 14 | The Propagation of Waves in Elastic Solid Media 485

165. Introduction **485**
166. Waves of Dilatation and Waves of Distortion in Isotropic Elastic Media **486**
167. Plane Waves **487**
168. Longitudinal Waves in Uniform Bars. Elementary Theory **492**
169. Longitudinal Impact of Bars **497**
170. Rayleigh Surface Waves **505**
171. Spherically Symmetric Waves in the Infinite Medium **508**
172. Explosive Pressure in a Spherical Cavity **510**

Appendix | The Application of Finite-difference Equations in Elasticity 515

1. Derivation of Finite Difference Equations **515**
2. Methods of Successive Approximation **520**
3. Relaxation Method **522**
4. Triangular and Hexagonal Nets **527**
5. Block and Group Relaxation **532**
6. Torsion of Bars with Multiply-connected Cross Sections **534**
7. Points Near the Boundary **536**
8. Biharmonic Equation **538**
9. Torsion of Circular Shafts of Variable Diameter **545**
10. Solutions by Digital Computer **549**

Name Index 553

Subject Index 559