

# Contents

Preface ix

Straight Forward to the Student xiii

Table of Symbols xv

## CHAPTER 1

### Manifolds and Smooth Maps 1

- §1 DEFINITIONS 1
- §2 DERIVATIVES AND TANGENTS 8
- §3 THE INVERSE FUNCTION THEOREM AND, IMMERSIONS 13
- §4 SUBMERSIONS 20
- §5 TRANSVERSALITY 27
- §6 HOMOTOPY AND STABILITY 33
- §7 SARD'S THEOREM AND MORSE FUNCTIONS 39
- §8 EMBEDDING MANIFOLDS IN EUCLIDEAN SPACE 48

CHAPTER **2****Transversality and Intersection 57**

- §1 MANIFOLDS WITH BOUNDARY 57
- §2 ONE-MANIFOLDS AND SOME CONSEQUENCES 64
- §3 TRANSVERSALITY 67
- §4 INTERSECTION THEORY MOD 2 77
- §5 WINDING NUMBERS AND THE JORDAN-BROUWER SEPARATION THEOREM 85
- §6 THE BORSUK-ULAM THEOREM 91

CHAPTER **3****Oriented Intersection Theory 94**

- §1 MOTIVATION 94
- §2 ORIENTATION 95
- §3 ORIENTED INTERSECTION NUMBER 107
- §4 LEFSCHETZ FIXED-POINT THEORY 119
- §5 VECTOR FIELDS AND THE POINCARÉ-HOPF THEOREM 132
- §6 THE HOPF DEGREE THEOREM 141
- §7 THE EULER CHARACTERISTIC AND TRIANGULATIONS 148

CHAPTER **4****Integration on Manifolds 151**

- §1 INTRODUCTION 151
- §2 EXTERIOR ALGEBRA 153
- §3 DIFFERENTIAL FORMS 162
- §4 INTEGRATION ON MANIFOLDS 165
- §5 EXTERIOR DERIVATIVE 174
- §6 COHOMOLOGY WITH FORMS 178
- §7 STOKES THEOREM 182
- §8 INTEGRATION AND MAPPINGS 188
- §9 THE GAUSS-BONNET THEOREM 194

APPENDIX 1

Measure Zero and Sard's Theorem 202

APPENDIX 2

Classification of Compact One-Manifolds 208

Bibliography 212

Index 217