PREFACE

1 INTRODUCTION

Summary of the Chapters 3 Notation 10 Teaching a Course from This Book 13 The Problems 13 Aspects of Imaging 15 Computer Code 17 Literature References 17 Recommendation 19

2 THE IMAGE PLANE

Modes of Representation 21 Some Properties of a Function of Two Variables 40 Projection of Solid Objects 43 Image Distortion 50 Operations in the Image Plane 67 xiii

1

20

V

Binary Images 77 Operations on Digital Images 80 Reflectance Distribution 83 Data Compression 89 Summary 91 Appendix: A Contour Plot Program 91 Literature Cited 95 Further Reading 97 Problems 97

3 TWO-DIMENSIONAL IMPULSE FUNCTIONS

111

The Two-Dimensional Point Impulse 111 Rules for Interpreting Delta Notation 115 Generalized Functions 117 The Shah Functions III and ²III 117 Line Impulses 119 Regular Impulse Patterns 122 Interpretation of Rectangle Function of f(x) 123 Interpretation of Rectangle Function of f(x,y)125 General Rule for Line Deltas 125 The Ring Impulse 128 Impulse Function of f(x,y) = 130Sifting Property 131 Derivatives of Impulses 133 Summary 135 Literature Cited 135 Problems 136

4

THE TWO-DIMENSIONAL FOURIER TRANSFORM 140

One Dimension 141 The Fourier Component in Two Dimensions 141 Three or More Dimensions 143 Vector Form of Transform 143 The Corrugation Viewpoint 144 Examples of Transform Pairs 147 Theorems for Two-Dimensional Fourier Transforms 154 The Two-Dimensional Hartley Transform 166

vi

Discrete Transforms 167 Summary 168 Literature Cited 169 Further Reading 169 Problems 169

5 TWO-DIMENSIONAL CONVOLUTION

Convolution Defined 176 Cross-Correlation Defined 179 Feature Detection by Matched Filtering 180 Autocorrelation Defined 181 Understanding Autocorrelation 183 Cross-Correlation Islands and Dilation 187 Lazy Pyramid and Chinese Hat Function 187 Central Value and Volume of Autocorrelation 192 The Convolution Sum 193 Computing the Convolution 195 Digital Smoothing 196 Matrix Product Notation 196 Summary 199 Literature Cited 200 Problems 200

6 THE TWO-DIMENSIONAL CONVOLUTION THEOREM

Convolution Theorem 206 An Instrumental Caution 207 Point Response and Transfer Function 208 Autocorrelation Theorem 208 Cross-Correlation Theorem 209 Factorization and Separation 210 Convolution with the Hartley Transform 211 Summary 214 Problems 215

7 SAMPLING AND INTERPOLATION IN TWO DIMENSIONS

What is a Sample? 223 Sampling at a Point 224 174

204

Sampling on a Point Pattern, and the Associated Transfer Function 224 Sampling Along a Line 227 Curvilinear Sampling 227 The Shah Function 228 Fourier Transform of the Shah Function 229 Other Patterns of Sampling 230 Factoring 234 The Two-Dimensional Sampling Theorem 235 Undersampling 240 Aliasing 241 Circular Cutoff 243 Double-Rectangle Pass Band 245 Discrete Aspect of Sampling 246 Interpolating Between Samples 247 Interlaced Sampling 254 Appendix: The Two-Dimensional Fourier Transform of the Shah Function 257 Literature Cited 259 Problems 260

8 DIGITAL OPERATIONS

Smoothing 267 Nonconvolutional Smoothing 271 Trend Reduction 273 Sharpening 274 What is a Digital Filter? 276 Guard Zone 276 Transform Aspect of Smoothing Operator 277 Finite Impulse Response (FIR) 278 Special Filters 279 Densifying 283 The Arbitrary Operator 284 Derivatives 285 The Laplacian Operator 287 Projection as a Digital Operation 287 Moiré Patterns 290 Functions of an Image 300

Digital Representation of Objects 301 Filling a Polygon 312 Edge Detection and Segmentation 314 Discrete Binary Objects 315 Operations on Discrete Binary Objects 318 Union and Intersection 319 Pixel Morphology 321 Dilation 322 Coding a Binary Matrix 335 Granulometry 336 Conclusion 337 Literature Cited 337 Problems 338

9 ROTATIONAL SYMMETRY

What Is a Bessel Function? 347 The Hankel Transform 352 The jinc Function 359 The Struve Function 365 The Abel Transform 365 Spin Averaging 371 Angular Variation and Chebyshev Polynomials 375 Summary 379 Table of the jinc Function 381 Problems 381

10 IMAGING BY CONVOLUTION

Mapping by Antenna Beam 387 Scanning the Spherical Sky 392 Photography 394 Microdensitometry 395 Video Recording 397 Eclipsometry 397 The Scanning Acoustic Microscope 398 Focusing Underwater Sound 400 Literature Cited 400 Problems 401

あます きたわけ こうてい かいのう 語っていたい なくの おおき ひとうかん たいやく くってん

11 DIFFRACTION THEORY OF SENSORS AND RADIATORS

The Concept of Aperture Distribution 403 Source Pair and Wave Pair 404 Two-Dimensional Apertures 409 Rectangular Aperture 409 Example of Circular Aperture 410 Duality 411 The Thin Lens 412 What Happens at a Focus? 416 Shadow of a Straight Edge 420 Fresnel Diffraction in General 425 Literature Cited 427 Problems 427

12 APERTURE SYNTHESIS AND INTERFEROMETRY 430

Image Extraction from a Field 430 Incoherent Radiation Source 431 Field of Incoherent Source 432 Correlation in the Field of an Incoherent Source 435 Visibility 438 Measurement of Coherence 439 Notation 440 Interferometers 441 Radio Interferometers 443 Rationale Behind Two-Element Interferometer 445 Aperture Synthesis (Indirect Imaging) 447 Literature Cited 449 Problems 450

13 RESTORATION

Restoration by Successive Substitutions 454 Running Means 456 Eddington's Formula 458 Finite Differences 460 Finite Difference Formula 461 Chord Construction 462 The Principal Solution 464

х

Finite Differencing in Two Dimensions 468 Restoration in the Presence of Errors 472 The Additive Noise Signal 474 Determination of the Real Restoring Function 477 Determination of the Complex Restoring Function 479 Some Practical Remarks 479 Artificial Sharpening 481 Antidiffusion 482 Nonlinear Methods 485 Restoring Binary Images 485 CLEAN 485 Maximum Entropy 486 Literature Cited 487 Problems 489

14 THE PROJECTION-SLICE THEOREM

Circular Symmetry Reviewed 494 The Abel-Fourier-Hankel Cycle 495 The Projection-Slice Theorem 498 Literature Cited 501 Problems 503

15 COMPUTED TOMOGRAPHY

Working from Projections 506 An X-Ray Scanner 508 Fourier Approach to Computed Tomography 510 Back-Projection Methods 512 The Radon Transform 517 The Impulse Response of the Radon Transformation 518 Some Radon Transforms 526 The Eigenfunctions 530 Theorems for the Radon Transform 530 The Radon Boundary 532 Applications 533 Literature Cited 536 Problems 538

16	SYNTHETIC-APERTURE RADAR	545
	Doppler Radar 545 Some History of Radiofrequency Doppler 550 Range-Doppler Radar 551 Radargrammetry 557 Literature Cited 558 Problems 558	
17	TWO-DIMENSIONAL NOISE IMAGES	560
	Some Types of Random Image 561 Gaussian Noise 582 The Spatial Spectrum of a Random Scatter 586 Autocorrelation of a Random Scatter 591 Pseudorandom Scatter 593 Random Orientation 599 Nonuniform Random Scatter 599 Spatially Correlated Noise 601 The Familiar Maze 604 The Drunkard's Walk 607 Fractal Polygons 609 Conclusion 618 Literature Cited 619 Problems 620	
APPENDIX A SOLUTIONS TO PROBLEMS		623
	INDEX	676

xii