# Contents

## Preface xi

## **Chapter One**

## **Basic Concepts**

| §1 | The Combinatorial Method 1              |
|----|---|
| §2 | Continuous Transformations in the Plane |
| §3 | Compactness and Connectedness 22        |
| 84 | Abstract Point Set Topology 28          |
|    |   |

11

# Chapter Two

# Vector Fields

**§5** A Link Between Analysis and Topology 33

#### **VIII CONTENTS**

- 96 Sperner's Lemma and the Brouwer Fixed Point Theorem 36
- **§7** Phase Portraits and the Index Lemma 43
- **Winding Numbers 48**
- 99 Isolated Critical Points 54
- §10 The Poincaré Index Theorem 60
- §11 Closed Integral Paths 67
- **§12** Further Results and Applications 73

#### **Chapter Three**

#### Plane Homology and the Jordan Curve Theorem

- 13 Polygonal Chains 79
- 1.4 The Algebra of Chains on a Grating 84
- 15 The Boundary Operator 88
- **§16** The Fundamental Lemma 91
- 17 Alexander's Lemma 97
- **§18** Proof of the Jordan Curve Theorem 100

#### **Chapter Four**

## Surfaces

1 9 Examples of Surfaces 104\$20 The Combinatorial Definition of a Surface 116

4

- 2.1 The Classification Theorem 122
- 22 Surfaces with Boundary 129

## **Chapter Five**

#### **Homology of Complexes**

- 2 3 Complexes 132
- **§24** Homology Groups of a Complex 143
- 2.5 Invariance 153
- 2.6 Betti Numbers and the Euler Characteristic 159
- 2.7 Map Coloring and Regular Complexes 169
- 2.8 Gradient Vector Fields 176
- 2 9 Integral Homology 185
- 30 Torsion and Orientability 192
- 31 The Poincaré Index Theorem Again 200

#### **Chapter Six**

#### **Continuous** Transformations 32 Covering Spaces 209 Simplicial Transformations 221 33 Invariance Again 228 34 35 Matrixes 234 The Lefschetz Fixed Point Theorem 36 242 Homotopy 251 37 38 Other Homologies 259

## **X** CONTENTS

## Supplement

# **Topics in Point Set Topology**

| 39    | Cryptomorphic Versions of Topology 26                               | 5   |
|-------|---|-----|
| 4 0   | A Bouquet of Topological Properties 27                              | 3   |
| 41    | Compactness Again 279   |     |
| §42   | Compact Metric Spaces 284   |     |
|       |   |     |
| Hints | and Answers for Selected Problems                                   | 287 |
|       | and Answers for Selected Problems<br>stions for Further Reading 302 | 287 |

Index 305