CONTENTS

PREFACE TO THE SERIES viii

PREFACE ix

CHAPTER 1 STATISTICAL MECHANICS AND QUANTUM FIELD THEORY 1
1.1 Quantum Particles 1
1.2 Global and Local Symmetries. Preliminary Description 4
1.3 Discrete Global Symmetries 5
1.4 Continuum Abelian Global Symmetries 11
1.5 Non-Abelian Global Symmetries 13
1.6 Discrete Gauge Symmetries 14
1.7 O(2) Gauge Systems 15
1.8 Non-Abelian Gauge Theories 17

CHAPTER 2 ASYMPTOTIC FREEDOM AND THE RENORMALIZATION GROUP 19
2.1 Principal Chiral Fields 19
2.2 The n-Fields 27
2.3 Non-Abelian Gauge Fields for $\mathcal{N} = 4$ 29

CHAPTER 3 THE STRONG COUPLING EXPANSION 33
3.1 Ising Model 34
3.2 Continuous Global Symmetry 37
3.3 Gauge Symmetries 40

CHAPTER 4 INSTANTONS IN ABELIAN SYSTEMS 49
4.1 Instantons in Quantum Mechanics and the Ising model 49
4.2 Instantons in the Global O(2) Model 54
4.3 Compact QED (O(2) Gauge Model) 62

CHAPTER 5 QUARK CONFINEMENT, SUPERFLUIDITY, ELASTICITY. CRITERIA AND ANALOGIES 73

CHAPTER 6 TOPOLOGY OF GAUGE FIELDS AND RELATED PROBLEMS 85
6.1 Instantons for $\mathscr{D} = 2$, $N = 3n$-Fields 85
6.2 Instantons in Non-Abelian Gauge Theories 92
6.3 Qualitative Effects of Instantons 99

CHAPTER 7 ANALOGIES BETWEEN GAUGE AND CHIRAL FIELDS. LOOP DYNAMICS 111
7.1 Non-Abelian Phase Factor 111
7.2 Quantum Theory of Loops 119

CHAPTER 8 THE LARGE N EXPANSION 125
8.1 O(N) a-Model 125
8.2 The Principal Chiral Field for $SU(N)$ 134
8.3 The $CP^{N-1}n$-model 139
8.4 Non-Abelian Gauge Theory 144

CHAPTER 9 QUANTUM STRINGS AND RANDOM SURFACES 151
9.1 Mathematical Preliminaries: Summation of Random Paths 151
9.2 Measures in the Space of Metrics and Diffeomorphisms 157
9.3 Closed Paths 164
9.4 General Theory of Random Hypersurfaces 169
9.5 Two-Dimensional Surfaces. Geometrical Introduction 176
9.6 Computation of Functional Integrals 185
9.7 Scattering Amplitudes 192
9.8 Scattering Amplitudes and the Operator Product Expansion 195
9.9 The Energy-Momentum Tensor in Conformal Quantum Field Theory 203
9.10 Physical States of String Theory in the Critical Dimension 212
9.11 Fermi Particles 222

9.12 Fermionic Strings 228
9.13 Vertex Operators 240

CHAPTER 10 ATTEMPT AT A SYNTHESIS 253
10.1 Long Wave Oscillations of Strings in Critical Dimensions 253
10.2 Possible Applications of Critical Strings 266
10.3 The Three Dimensional Ising Model 273
10.3.1 The Dirac Equation in the Two Dimensional Ising Model 275
10.3.2 The Three Dimensional Case. The Loop Equation 278
10.4 Extrinsic Geometry of Strings 283

SUBJECT INDEX 289