CONTENTS

Lecturers ix

Participants xi

Préface xv

Preface xix

Contents xxiii

Course 1. Applied *conformal* field ***theory, by*** P. Ginsparg 1

1. Conformal theories in d dimensions 6
   1.1. Conformal group in d dimensions 6
   1.2. Conformal algebra in 2 dimensions 8
   1.3. Constraints of conformal invariance in d dimensions 11
2. Conformal theories in 2 dimensions 14
   2.1. Correlation functions of primary fields 14
   2.2. Radial quantization and conserved charges 17
   2.3. Free boson, the example 22
   2.4. Conformal Ward identities 25
3. The central charge and the Virasoro algebra 27
   3.1. The central charge 27
   3.2. The free fermion 30
   3.3. Mode expansions and the Virasoro algebra 31
   3.4. In- and out-states 32
   3.5. Highest weight states 36
   3.6. Descendant fields 39
   3.7. Duality and the bootstrap 42
4. Kac determinant and unitarity 45
   4.1. The Hilbert space of states 45
   4.2. Kac determinant 49

4.3. Sketch of non-unitarity proof 51
4.4. Critical statistical mechanical models 53
4.5. Conformal grids and null descendants 56
5. Identification of $m = 3$ with the critical Ising model 57
5.1. Critical exponents 58
5.2. Critical correlation functions of the Ising model 61
5.3. Fusion rules for $c < 1$ models 64
5.4. More discrete series 66
6. Free bosons and fermions 69
6.1. Mode expansions 69
6.2. Twist fields 70
6.3. Fermionic zero modes 74
7. Free fermions on a torus 77
7.1. Back to the cylinder, on to the torus 77
7.2. $c = \frac{1}{2}$ representations of the Virasoro algebra a2
7.3. The modular group and fermionic spin structures 84
7.4. $c = \frac{1}{2}$ Virasoro characters 88
7.5. Critical Ising model on the torus 91
7.6. Recreational mathematics and ϑ-function identities 97
8. Free bosons on a torus 104
8.1. Partition function 104
8.2. Fermionization 110
8.3. Orbifolds in general 112
8.4. S^1/Z_2 orbifold 116
8.5. Orbifold comments 121
8.6. Marginal operators 123
8.7. The space of $c = 1$ theories 125
9. Affine Kac-Moody algebras and coset constructions 130
9.1. Affine algebras 130
9.2. Enveloping Virasoro algebra 132
9.3. Highest weight representations 136
9.4. Some free field representations 141
9.5. Coset construction 145
9.6. Modular invariant combinations of characters 150
9.7. The A-D-E classification of SU(2) invariants 153
9.8. Modular transformations and fusion rules 157
10. Advanced applications 158
References 159

Course 2. John L. Cardy, Conformal invariance and statistical mechanics 169

1. Introduction 173
1.1. Outline and aims 173
1.2. Statistical mechanics and quantum field theory 174
1.3. Scale invariance, conformal invariance and the stress tensor 177
1.4. The Gaussian model 182
2. Finite-size scaling of the free energy 184
2.1. Finite-size scaling at criticality 184
2.2. The trace anomaly 185
2.3. Systems with a boundary 188
2.4. Corners on the boundary 191
2.5. On hearing the shape of a drum 193
3. Theories defined on a cylinder 194
3.1. Transfer matrix on the cylinder 194
3.2. Lattice models on the cylinder 197
3.3. Other boundary conditions 202
4. Modular invariance on the torus 204
4.1. Theories defined on a torus 204
4.2. The Gaussian model 296
4.3. Decomposition into characters 297
4.4. Theories with c < 1 268
5. Identification of operators in particular models 212
5.1. The fusion rules 212
5.2. The Ising model 213
5.3. Lattice effects 218
5.4. Landau-Ginzburg classification 218
5.5. The J-state Potts model 220
6. The continuum limit away from criticality 221
6.1. The stress tensor 221
6.2. Zamolodchikov's c-theorem 223
6.3. The Ising model 226
6.4. Perturbative calculation of c 227
6.5. Application to models with c < 1 229
6.6. Conserved currents away from criticality 231
7. The spectrum of the corner transfer matrix in solvable models 234
7.1. Introduction 234
7.2. Commuting transfer matrices 235
7.3. Corner transfer matrix 237
7.4. The eigenvalues of $\hat{A}(u)$ 249

7.5. Character formulae 241
References 243

Course 3. *Conformal* field theories, ***Coulomb gas picture and*** integrable models, ***by*** J.-B. Zuber 247

1. Introduction 250
2. c = 1 conformal theories and the 6-vertex model 250
3. c < 1 conformal theories 254
3.1. Classification of minimal theories 254
3.2. Feigin-Fuchs representation 259
3.3. Coulomb gas picture in Statistical Mechanics 260
3.4. Floating charges on a torus 262
3.5. Restricted Solid-on-Solid models 265
4. Generalized height models 266
4.1. Yang-Baxter equation and Temperley-Lieb algebra 266
4.2. Models attached to graphs 269
5. Higher spin models 273
5.1. Higher vertex models and their critical line 273
5.2. Higher $SU(2)\times SU(2)/SU(2)$ coset models 275
6. Conclusion 276
References 277

Seminar 1. Symmetries of the XXZ chain and ***quantum*** *SU(2)*, by V. *Pasquier* and H. *Saleur* 281

Seminar 2. Recent progress in rational *conformal* ***field theory,*** by R. Dijkgraaf 291

1. Introduction 294
2. Rational conformal field theories 294
3. RCFT in genus one 298
4. An example 301
5. Rational Gaussian models 302
References 303

Course 4. Two-dimensional *quantum gravity. Superconductivity at high T_C, by A. Polyakov* 305

I. Introduction 308
1.1. Aim of the lectures 308
1.2. Elementary excitations 308
1.3. Relation with statistical mechanics 311
1.4. String-like excitations 314
2. Particles with spin and path integral representation 315
3. Spin factor and fermionic integral 324
4. 2D surfaces and strings 328
5. Fermionic determinants 333
6. Dynamical gauge fields 339
7. Two-dimensional quantum gravity 349
8. More on 2D quantum gravity 351
9. High T_c superconductivity 358
10. Anomalous dimensions of operators 363

Course 5. Exactly solvable models of 2D-quantum gravity on the lattice, by V.A. Kazakov 369

1. Lattice formulation of 2D-quantum gravity with various matter fields and review of the results 373
2. Statistical models on the dynamically triangulated planar lattice (DPL-models) 382
References 391

Seminar 3. Conformal invariance, ***self-avoiding walks in the plane or on a random surface, by B. Duplantier*** 393

1. Introduction 395
2. Polymers in the plane 396
2.1. Watermelon topology 396
2.2. $O(n)$ model and Coulomb gas technique 398
3. Polymers on a two-dimensional random lattice 400
3.1. The model 400
3.2. Critical behaviors 403
4. Conformal invariance on a random surface 404
5. Higher topologies 406
References 407

Course 6. Some topics in string theory, by A. Neveu 409

1. Introduction 413
2. Descriptive properties of strings 414
2.1. Basic ingredients of free bosonic strings 414
2.2. Basic ingredients of free superstrings 416
3. String Interactions 418
3.1. Elementary vertex operators 418
3.2. The Koba-Nielsen formula and its interpretation 419
3.3. Covariant quantization of the bosonic string 420
4. String field theory 423
4.1. Motivation 423
4.2. Light-cone string field theory 423
4.3. Lorentz covariant free string field theory 424
4.4. Interacting gauge covariant strings 428
4.5. Superstring field theories 430
5. Group theoretic approach to strings 431
5.1. Motivation and overview 431
5.2. Overlaps 432
5.3. Determination of the integration measure 434
5.4. String vertices as induced representations 436
References 437

Course 7. Space-time supersymmetry, effective Lagrangians, and the moduli space of superconformal field theories, by S. Ferrara . 441

Summary of the Lectures 443
References 447

Course 8. ***Selected*** topics in lattice field theory, by M. Luscher . 451

1. Introduction 455
2. Particle scattering in euclidean lattice theories 457
2.1. Construction and interpretation of the transfer matrix 457
2.2. Localized one-particle states 461
2.3. Scattering states 467
2.4. LSZ type formula for the scattering matrix 469
3. Renormalization and continuum limit in perturbation theory 474
3.1. Examples and introductory remarks 474
3.2. Structure of lattice Feynman integrals 477
3.3. Degrees of divergence 480
3.4. The Reisz power counting theorem 482

3.5. Gauge fixing and BRS symmetry on the lattice 484
Appendix 1. Proof of the lattice power counting theorem for $L = 1$ 494
4. Finite size effects in massive theories 497
4.1. Basic facts 497
4.2. Volume dependence of stable particle masses in simple models 500
4.3. Models with a spontaneously broken discrete symmetry 503
4.4. The case of (pure) non-Abelian gauge theories 507
4.5. Two-particle states in finite volume 515
4.6. Resonances 519
References 525

Course 9. Principles of numerical simulations, by G. Parisi . . . 529

1. Introduction 533
2. The floating numbers 534
3. Random numbers 535
4. Solving differential equations 538
5. Simulating statistical mechanics 540
6. Preliminaries on field theories 543
7. Pure gauge QCD: the spectrum 545
8. The deconfinement transition in pure gauge QCD 552
9. Quenched fermions 556
10. Open problems 560
References 561

Course 10. Field theory methods and quantum critical

1. Introduction 567
2. Spin-wave theory for the Heisenberg model [1] 569
3. Large-s mapping onto the O(3) non-linear g-models [5] 575
4. Behavior of the O(3) non-linear a-model: hints from the renormalization group, large-n limits and exact S-matrix 579
5. The Lieb-Shultz-Mattis theorem 585
6. Spin-1/2 chain by Jordan-Wigner transformation 588
7. The Hubbard model-large-U limit and spin-density-wave mean field theory 600
8. Non-abelian bosonization of the Hubbard model 606
9. Higher spin, Bethe ansatz and finite-size scaling 614
Appendix 1. Notation, free fermion and free boson field theories 634
References 639

Course 11. Strings . . . . by Dan Friedan 641